Name of College – S.S. College Jehanabad.

Deptt – Mathematics

Topic – Polar Sub-tangent and Subnormal (Differential Calculus)

Date – 16-04-2020 Time – 10:15 to 11:45

Class – B.Sc I (Hons)

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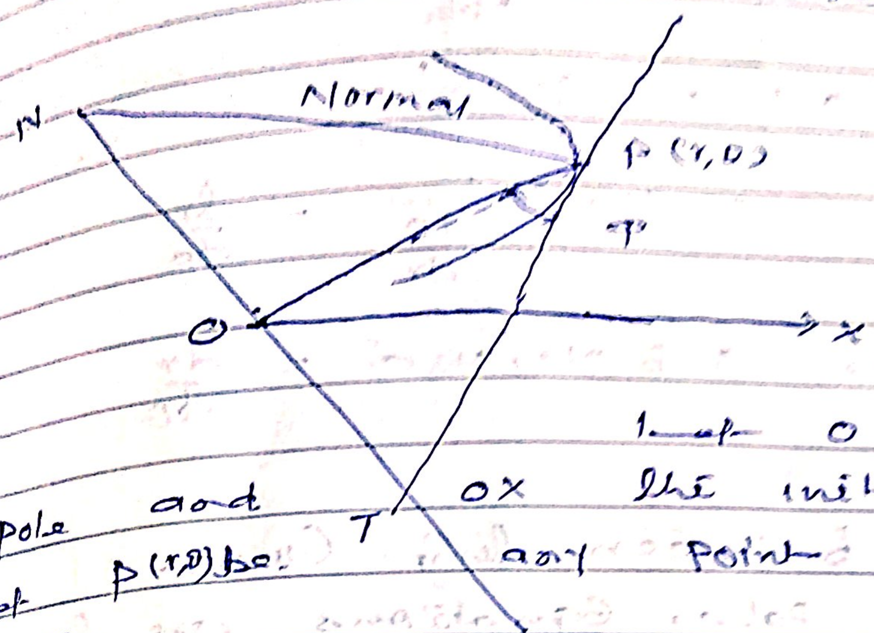

Let O be the pole and OX the initial line. Let P(r,θ) be any point on the curve.

PT and PN be the Normal drawn at P. From O st. line NOT is drawn which is perpendicular to OP.

Let this line meets the Tangent at T and Normal at the point N. Then OT is called the polar sub-tangent and ON is polar sub-normal.

Let $\angle OPT = \varphi$

then from ΔPOT $\tan\varphi = \frac{OT}{OP}$

$$\therefore OT = OP\tan\varphi$$

$$= r\tan\varphi$$

$$= r \cdot r\frac{d\theta}{dr}$$

$$= r^2\frac{d\theta}{dr}$$

Therefore,

polar Subtangent $= r^2 \frac{d\theta}{dr}$

Again since $\angle OPT = \varphi$

$\therefore \angle OPN = 90 - \varphi$

$\Rightarrow \angle ONP = \varphi$

From $\triangle OPT$

$$\tan\varphi = \frac{OP}{ON}$$

$$ON = OP \cdot \frac{1}{\tan\varphi}$$

$$= r \cdot \frac{1}{r\frac{d\theta}{dr}} = \frac{dr}{d\theta}$$

$\therefore$ Polar Sub Normal $= \frac{dr}{d\theta}$

Angle between two curves whose polar equations are given.

Let the polar equations of two curves be

$r = f(\theta)$ and $r = g(\theta)$

Let P = point of intersection of these two curves

Since the angle of intersection of two curves is same as the angle between their tangent at their point of intersection

Let the angle between radius ~~vector~~ ~~and~~ and Tangents at these point be φ_1 and φ_2

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$\Rightarrow \tan\varphi_1 = r\dfrac{d\theta}{dr} = \dfrac{f(\theta)}{f'(\theta)}$

$\tan\varphi_2 = r\dfrac{d\theta}{dr} = \dfrac{g(\theta)}{g'(\theta)}$

If α be the angle between them then

$$\alpha = \varphi_1 - \varphi_2$$

$$\Rightarrow \tan\alpha = \tan(\varphi_1 - \varphi_2)$$

$$= \frac{\tan\varphi_1 - \tan\varphi_2}{1 + \tan\varphi_1 \cdot \tan\varphi_2}$$

$$= \frac{\dfrac{f(\theta)}{f'(\theta)} - \dfrac{g(\theta)}{g'(\theta)}}{1 + \dfrac{f(\theta)}{f'(\theta)} \cdot \dfrac{g(\theta)}{g'(\theta)}}$$

$$= \frac{f(\theta)\, g'(\theta) - g(\theta)\, f'(\theta)}{f(\theta) \cdot g(\theta) + f'(\theta) \cdot g'(\theta)}$$

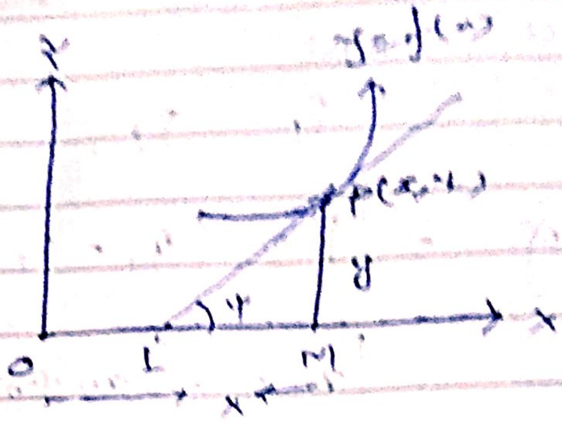
$\alpha = 0 \Rightarrow$ the given two curves touch each other

$\therefore f(\theta)\, g'(\theta) - g(\theta)\, f'(\theta) = 0$

If $\alpha = \pi/2$

$\Rightarrow f'(\theta) \cdot g'(\theta) + f(\theta) \cdot g(\theta) = 0$

Problem 2 :- Show that in the curve $y = a\log(x^2 - a^2)$, the sum of the tangent and subtangent varies as the product of the co-ordinates of the point.



From $\triangle LMP$

Since $\dfrac{MP}{LP} = \sin\psi$

$$\Rightarrow LP = \frac{MP}{\sin\psi} = y\,\mathrm{cosec}\,\psi$$

Length of tangent

and $\dfrac{MP}{LM} = \tan\psi$

$$\Rightarrow LM = \frac{MP}{\tan\psi} = \frac{y}{y_1}$$

$\Rightarrow$ Subtangent $= \dfrac{y}{y_1}$ where $y_1 = \left(\dfrac{dy}{dx}\right)$

Since Equation of curve is

$$y = a\log(x^2 - a^2)$$

$$\frac{dy}{dx} = \frac{a}{x^2 - a^2} \times 2x$$

$$y_1 = \frac{2ax}{x^2 - y^2}$$

$$\therefore \mathrm{cosec}\,\psi = \sqrt{1 + \cot^2\psi}$$

$$= \sqrt{1 + \frac{1}{\tan^2\psi}}$$

$$= \sqrt{\frac{1 + \tan^2\psi}{\ldots}} = \boxed{1 + y_1^2}$$

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$\frac{1}{y_1}\sqrt{1+y_1^2}$

length of tangent $= y\,\text{cosec}\,\psi$

$= \frac{y}{y_1}\sqrt{1+y_1^2}$

length of subtangent $= \frac{y}{y_1}$

$\therefore$ seem of Tangent and sub-tangent

$= \frac{y\sqrt{1+y_1^2}}{y_1} + \frac{y}{y_1}$

$= \frac{y\sqrt{1+y_1^2}+y}{y_1}$

$= \frac{y\left[\sqrt{1+y_1^2}+1\right]}{y_1}$

$= \frac{y\left[\sqrt{1+\frac{4a^2x^2}{(x^2-a^2)^2}}+1\right]}{\frac{2ax}{x^2-a^2}}$

$= \frac{y\left[\frac{\sqrt{(x^2-a^2)^2+4a^2x^2}}{x^2-a^2}+1\right]}{\frac{2ax}{x^2-a^2}}$

$= \frac{y\left[x^2+\cancel{a^2}+x^2-\cancel{a^2}\right]}{(\cancel{x^2-a^2})\times\frac{2ax}{\cancel{x^2-a^2}}}$

$= \frac{y\times\cancel{2}x^{\cancel{2}}}{\cancel{2}a\cancel{x}} = \frac{xy}{a}$

Thus Sum of tangent and Subtangent $= \frac{1}{a} = \text{const}$

$\overline{xy}$

$\Rightarrow$ Sum of tangent and subtangent $\propto xy$

Hence the result.

Problem $\rightarrow$

Find the angle of intersection of curves

$r = a(1+\cos\theta) = f(\theta)$ —— (i)

$r = b(1-\cos\theta) = g(\theta)$ —— (ii)

From (i)

$r = a(1+\cos\theta)$

$\log r = \log a + \log(1+\cos\theta)$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{1}{1+\cos\theta}(-\sin\theta)$

$= -\frac{\sin\theta}{1+\cos\theta}$

$= -\frac{2\sin\theta/2\cos\theta/2}{2\cos^2\theta/2}$

$= -\tan\theta/2$

$\therefore r\frac{d\theta}{dr} = -\cot\theta/2$

$\Rightarrow \tan\phi_1 = -\cot\theta/2$

From (ii)

$r = b(1-\cos\theta)$

Taking log on both sides

$\log r = \log b + \log(1-\cos\theta)$

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = \frac{1}{1-\cos\theta} \times \sin\theta$$

$$= \frac{2\sin\theta/2\cos\theta/2}{2\sin^2\theta/2}$$

$$= \cot\theta/2$$

$$r\frac{d\theta}{dr} = \tan\theta/2$$

$$\tan\phi_2 = \tan\theta/2$$

~~Here $\phi = \phi_1 - \phi_2$~~

Here $\tan\phi_1 \cdot \tan\phi_2 = -\cot\theta/2 \times \tan\theta/2$
$= -1$

Hence the two curves cut each other at $\pi/2$.

Prove that the curves $r^m = a^m \cos m\theta$ and $r^m = b^m \sin m\theta$ cut orthogonally.

Here $r^m = a^m \cos m\theta$

$m \log r = m \log a + \log\cos m\theta$

$$\frac{m}{r}\cdot\frac{dr}{d\theta} = \frac{1}{\cos m\theta}(-m\sin m\theta)$$

$$= -\tan m\theta$$

$$\frac{1}{r}\frac{dr}{d\theta}$$

$$r\frac{d\theta}{dr} = -\cot m\theta = \tan(\pi/2 + m\theta)$$

$$\tan\phi_1 = \tan(\pi/2 + m\theta)$$

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$\Rightarrow \phi_1 = \pi/2 + m\theta$

Again From (ii)

$r^m = b^m \sin m\theta$

$m \log r = m \log b + \log \sin m\theta$

$\frac{m}{r} \frac{dr}{d\theta} = 0 + \frac{1}{\sin m\theta} \times m \cos m\theta$

$= \cot m\theta$

$r \frac{d\theta}{dr} = \frac{1}{\cot m\theta}$ $\tan m\theta$

$\therefore \phi_2 = m\theta$

Now angle between the curves

= the angle of intersection

$= \phi_1 - \phi_2$

$= \pi/2 + m\theta - m\theta$

$= \pi/2$

Shows that the given curves intersect orthogonally